

SYLLABUS
MATHEMATICS (041)
CLASS XII 2012-13

Annexure - ' F '

One Paper	Three Hours	Marks: 100
Units		Marks
I. RELATIONS AND FUNCTIONS		10
II. ALGEBRA		13
III. CALCULUS		44
IV. VECTORS AND THREE - DIMENSIONAL GEOMETRY		17
V. LINEAR PROGRAMMING		06
VI. PROBABILITY		10
Total		100

The Question Paper will include question(s) based on values to the extent of 5 marks.

UNIT I. RELATIONS AND FUNCTIONS

1. Relations and Functions : (10) Periods

Types of relations: reflexive, symmetric, transitive and equivalence relations. One to one and onto functions, composite functions, inverse of a function. Binary operations.

2. Inverse Trigonometric Functions: (12) Periods

Definition, range, domain, principal value branches. Graphs of inverse trigonometric functions. Elementary properties of inverse trigonometric functions.

UNIT-II: ALGEBRA

1. Matrices: (18) Periods

Concept, notation, order, equality, types of matrices, zero matrix, transpose of a matrix, symmetric and skew symmetric matrices. Addition, multiplication and scalar multiplication of matrices, simple properties of addition, multiplication and scalar multiplication. Non-commutativity of multiplication of matrices and existence of non-zero matrices whose product is the zero matrix (restrict to square matrices of order 2). Concept of elementary row and column operations. Invertible matrices and proof of the uniqueness of inverse, if it exists; (Here all matrices will have real entries).

2. Determinants: (20) Periods

Determinant of a square matrix (up to 3 x 3 matrices), properties of determinants, minors, cofactors and applications of determinants in finding the area of a triangle. Adjoint and inverse of a square matrix. Consistency, inconsistency and number of solutions of system of linear equations by examples, solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix.

UNIT-III: CALCULUS

1. Continuity and Differentiability: (18) Periods

Continuity and differentiability, derivative of composite functions, chain rule, derivatives of inverse trigonometric functions, derivative of implicit functions. Concept of exponential and logarithmic functions.

Derivatives of logarithmic and exponential functions. Logarithmic differentiation, derivative of functions expressed in parametric forms. Second order derivatives. Rolle's and Lagrange's Mean Value Theorems (without proof) and their geometric interpretation.

2. Applications of Derivatives: (10) Periods

Applications of derivatives: rate of change of bodies, increasing/decreasing functions, tangents and normals, use of derivatives in approximation, maxima and minima (first derivative test motivated geometrically and second derivative test given as a provable tool). Simple problems (that illustrate basic principles and understanding of the subject as well as real-life situations).

3. Integrals: (20) Periods

Integration as inverse process of differentiation. Integration of a variety of functions by substitution, by partial fractions and by parts, simple integrals of the following type to be evaluated.

$$\int \frac{dx}{x^2 \pm a^2}, \int \frac{dx}{\sqrt{x^2 \pm a^2}}, \int \frac{dx}{\sqrt{a^2 - x^2}}, \int \frac{dx}{ax^2 + bx + c}, \int \frac{dx}{\sqrt{ax^2 + bx + c}}$$

$$\int \frac{px+q}{ax^2 + bx + c} dx, \int \frac{px+q}{\sqrt{ax^2 + bx + c}} dx, \int \sqrt{a^2 \pm x^2} dx, \int \sqrt{x^2 - a^2} dx$$

$$\int \sqrt{ax^2 + bx + c} dx, \int (px+q)\sqrt{ax^2 + bx + c} dx.$$

Definite integrals as a limit of a sum, Fundamental Theorem of Calculus (without proof). Basic properties of definite integrals and evaluation of definite integrals.

4. Applications of the Integrals: (10) Periods

Applications in finding the area under simple curves, especially lines, circles/parabolas/ellipses (in standard form only), Area between the two above said curves (the region should be clearly identifiable).

5. Differential Equations: (10) Periods

Definition, order and degree, general and particular solutions of a differential equation. Formation of differential equation whose general solution is given. Solution of differential equations by method of separation of variables, homogeneous differential equations of first order and first degree. Solutions of linear differential equation of the type:

$$\frac{dy}{dx} + py = q, \text{ where } p \text{ and } q \text{ are functions of } x \text{ or constant}$$

$$+ px = q, \text{ where } p \text{ and } q \text{ are functions of } y \text{ or constant}$$

UNIT-IV: VECTORS AND THREE-DIMENSIONAL GEOMETRY

1. Vectors: (12) Periods

Vectors and scalars, magnitude and direction of a vector. Direction cosines and direction ratios of a vector. Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio. Scalar (dot) product of vectors, projection of a vector on a line. Vector (cross) product of vectors. Scalar triple product of vectors.

2. Three - dimensional Geometry: $\frac{dx}{dy}$ (12) Periods

Direction cosines and direction ratios of a line joining two points. Cartesian and vector equation of a line, coplanar and skew lines, shortest distance between two lines. Cartesian and vector equation of a plane. Angle between (i) two lines, (ii) two planes. (iii) a line and a plane. Distance of a point from a plane.

UNIT-V: LINEAR PROGRAMMING

1. Linear Programming: (12) Periods

Introduction, related terminology such as constraints, objective function, optimization, different types of linear programming (L.P.) problems, mathematical formulation of L.P. problems, graphical method of solution for problems in two variables, feasible and infeasible regions, feasible and infeasible solutions, optimal feasible solutions (up to three non-trivial constraints).

UNIT-VI: PROBABILITY

1. Probability:

(18) Periods

Conditional probability, multiplication theorem on probability. independent events, total probability, Baye's theorem, Random variable and its probability distribution, mean and variance of random variable. Repeated independent (Bernoulli) trials and Binomial distribution.

Recommended Textbooks.

- 1) Mathematics Part I - Textbook for Class XI, NCERT Publication
- 2) Mathematics Part II - Textbook for Class XII, NCERT Publication

Weightage of marks over different dimensions of the question paper shall be as follows:

A. Weightage to different topics/content units

S.No.	Topics	Marks
1.	Relations and Functions	10
2.	Algebra	13
3.	Calculus	44
4.	Vectors & three-dimensional Geometry	17
5.	Linear programming	06
6.	Probability	10
	Total	100

Note: The Question Paper will include question(s) based on values to the extent of 5 marks.

B. Weightage to different forms of questions

S.No.	Forms of Questions	Marks for each question	No. of Questions	Total marks
1.	Very Short Answer questions (VSA)	01	10	10
2.	Short Answer questions (SA)	04	12	48
3.	Long answer questions (LA)	06	07	42
	Total		29	100

C. Scheme of Options

There will be no overall choice. However, internal choice in any four questions of four marks each and any two questions of six marks each has been provided.

D. Difficulty level of questions

S.No.	Estimated difficulty level	Percentage of marks
1.	Easy	15
2.	Average	70
3.	Difficult	15

Based on the above design, separate sample papers along with their blue prints and Marking schemes have been included in this document. About 20% weightage has been assigned to questions testing higher order thinking skills of learners.

**SAMPLE QUESTION PAPER
MATHEMATICS (041)
CLASS XII (2012-13)
BLUE PRINT**

S. No.	Topics	VSA	SA	LA	TOTAL
1.	(a) Relations and Functions	-	4 (1)	-	10 (4)
	(b) Inverse Trigonometric Functions	2 (2)	4 (1)	-	
2.	(a) Matrices	2 (2)	-	6 (1)	13 (5)
	(b) Determinants	1 (1)	4 (1)		
3.	(a) Continuity and Differentiability	1 (1)	12 (3)	-	44 (11)
	(b) Application of Derivatives	-	-	6 (1)	
	(c) Integration	-	12 (3)		
	(d) Applications of Integrals	-	-	6 (1)	
	(e) Differential Equations	1 (1)	-	6 (1)	
4.	(a) Vectors	2 (2)	4 (1)	-	17 (6)
	(b) 3-Dimensional Geometry	1 (1)	4 (1)	6 (1)	
5.	Linear Programming	-	-	6 (1)	6 (1)
6.	Probability	-	4 (1)	6 (1)	10 (2)
	Total	10 (10)	48 (12)	42 (7)	100 (29)

The Question Paper will include question(s) based on values to the extent of 5 marks.

Questionwise Analysis - Sample Paper

Question Number	Topics	Form of Question	Marks	Remarks
1	Inverse Trigonometric Functions	VSA	1	
2	Inverse Trigonometric Functions	VSA	1	
3	Matrices	VSA	1	
4	Matrices	VSA	1	
5	Determinants	VSA	1	
6	Differentiation	VSA	1	
7	Differential Equations	VSA	1	
8	Vectors	VSA	1	
9	Vectors	VSA	1	
10	3-Dimensional Geometry	VSA	1	
11	Relations and functions (Type of functions)	SA	4	
12	Inverse Trigonometric Functions	SA	4	
13	Determinants	SA	4	
14	Differentiation	SA	4	
15	Differentiation	SA	4	
16	Continuity of Functions	SA	4	
17	Integration * (Indefinite)	SA	4	
18	Integration * (Indefinite)	SA	4	
19	Integration * (Definite)	SA	4	
20	Vectors	SA	4	
21	3-Dimensional Geometry*	SA	4	
22	Probability	SA	4	
23	Matrices (Solution of System of equation)	LA	6	
24	Application of Derivative*	LA	6	
25	Application of Integration	LA	6	
26	Differential equations (Particular Solution)	LA	6	
27	3-Dimensional Geometry (Plane)	LA	6	
28	Linear Programming	LA	6	
29	Probability*	LA	6	

* With Alternative question

SAMPLE QUESTION PAPER
MATHEMATICS (041)
CLASS XII (2012-13)

General Instructions

1. All questions are compulsory.
2. The question paper consist of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 07 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators in not permitted. You may ask for logarithmic tables, if required.

SECTION - A

Question numbers 1 to 10 carry 1 mark each.

1. Using principal values, write the value of $2 \cos^{-1} \frac{1}{2} + 3 \sin^{-1} \frac{1}{2}$
2. Evaluate $\tan^{-1} [2\cos (2\sin^{-1} \frac{1}{2})]$
3. Write the value of $x+y+z$ if $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$
4. If A is a square matrix of order 3 such that $|\text{adj } A| = 225$, find $|A'|$
5. Write the inverse of the matrix $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$
6. The contentment obtained after eating x-units of a new dish at a trial function is given by the Function $C(x) = x^3+6x^2+5x+3$. If the marginal contentment is defined as rate of change of (x) with respect to the number of units consumed at an instant, then find the marginal contentment when three units of dish are consumed.
7. Write the degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 - 2\frac{d^2y}{dx^2} - \frac{dy}{dx} + 1 = 0$
8. If \vec{a} and \vec{b} are two vectors of magnitude 3 and $\frac{2}{3}$ respectively such that $\vec{a} \times \vec{b}$ is a unit vector, write the angle between \vec{a} and \vec{b} .
9. If $\vec{a} = 7\hat{i} + \hat{j} - 4\hat{k}$ and $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$, find the projection of \vec{a} on \vec{b}

10. Write the distance between the parallel planes $2x-y+3z=4$ and $2x-y+3z=18$

SECTION - B

Question numbers 11 to 22 carry 4 marks each.

11. Prove that the function $f: \mathbb{N} \rightarrow \mathbb{N}$, defined by $f(x) = x^2 + x + 1$ is one - one but not onto.

12. Show that $\sin[\cot^{-1}\{\cos(\tan^{-1}x)\}] = \frac{\sqrt{x^2+1}}{\sqrt{x^2+2}}$

OR

$$\text{Solve for } x: 3\sin^{-1}\left(\frac{2x}{1+x^2}\right) - 4\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + 2\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \frac{\pi}{3}$$

13. Two schools A and B decided to award prizes to their students for three values honesty (x), punctuality (y) and obedience (z). School A decided to award a total of Rs. 11000 for the three values to 5, 4 and 3 students respectively while school B decided to award Rs. 10700 for the three values to 4, 3 and 5 students respectively. If all the three prizes together amount to Rs. 2700, then.

- Represent the above situation by a matrix equation and form Linear equations using matrix multiplication.
- Is it possible to solve the system of equations so obtained using matrices?
- Which value you prefer to be rewarded most and why?

14. If $x = a(\theta - \sin \theta)$ and $y = a(1 - \cos \theta)$, find $\frac{d^2y}{dx^2}$

15. If $y = \frac{\sin^{-1}x}{\sqrt{1-x^2}}$, show that $(1-x^2)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y = 0$

16. The function $f(x)$ is defined as $f(x) = \begin{cases} x^2 + ax + b, & 0 \leq x < 2 \\ 3x + 2, & 2 \leq x \leq 4 \\ 2ax + 5b, & 4 < x \leq 8 \end{cases}$

If $f(x)$ is continuous on $[0,8]$, find the values of a and b

OR

Differentiate $\tan^{-1}\left[\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]$ with respect to $\cos^{-1}x^2$

17. Evaluate: $\int \frac{x^3+x+1}{x^2-1} dx$

OR

Evaluate: $\int e^x \frac{(1-\sin x)}{(1-\cos x)} dx$

18. Evaluate: $\int \frac{2x}{(x^2+1)(x^2+2)} dx$

19. Evaluate: $\int_0^{\frac{\pi}{4}} \log(1+\tan x) dx$, using properties of definite integrals

20. Let $\vec{a} = 4\hat{i} + 5\hat{j} - \hat{k}$, $\vec{b} = \hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} - \hat{k}$. Find a vector \vec{d} which is perpendicular to both \vec{a} and \vec{b} and satisfying $\vec{d} \cdot \vec{c} = 21$.

21. Find the distance between the point P(6, 5, 9) and the plane determined by the points A(3, -1, 2), B(5, 2, 4), and C(-1, -1, 6)

OR

Find the equation of the perpendicular drawn from the point P(2, 4, -1) to the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$. Also, write down the coordinates of the foot of the perpendicular from P to the line.

22. There is a group of 50 people who are patriotic out of which 20 believe in non violence. Two persons are selected at random out of them, write the probability distribution for the selected persons who are non violent. Also find the mean of the distribution. Explain the importance of Non violence in patriotism.

SECTION - C

Question numbers 23 to 29 carry 6 marks each

23. If $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$, find A^{-1} . Hence solve the following system of equations:

$$x+2y-3z = -4, 2x+3y+2z = 2, 3x-3y-4z = 11$$

24. Find the equations of tangent and normal to the curve $y = \frac{x-7}{(x-2)(x-3)}$ at the point where it cuts the x-axis

OR

Prove that the radius of the base of right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half that of the cone.

25. Find the area of the region enclosed between the two circles $x^2+y^2=1$ and $(x-1)^2 + y^2 = 1$

26. Find the particular solution of the differential equation:

$$(x-\sin y)dy + (\tan y)dx = 0 : \text{ given that } y=0 \text{ when } x=0$$

27. Find the vector and Cartesian equations of the plane containing the two lines

$$\vec{r} = 2\hat{i} + \hat{j} - 3\hat{k} + \lambda(\hat{i} + 2\hat{j} + 5\hat{k}) \text{ and } \vec{r} = 3\hat{i} + 3\hat{j} + 2\hat{k} + \mu(3\hat{i} - 2\hat{j} + 5\hat{k})$$

28. A dealer in rural area wishes to purchase a number of sewing machines. He has only Rs. 5760.00 to invest and has space for at most 20 items. A electronic sewing machine costs him Rs.360.00 and a manually operated sewing machine Rs. 240.00. He can sell an Electronic Sewing Machine at a profit of Rs. 22.00 and a manually operated sewing machine at a profit of Rs.18.00. Assuming that he can sell all the items that he can buy how should he invest his money in order to maximize his profit. Make it as a linear programming problem and solve it graphically. Keeping the rural background in mind justify the 'values' to be promoted for the selection of the manually operated machine.
29. In answering a question on a MCQ test with 4 choices per question, a student knows the answer, guesses or copies the answer. Let $\frac{1}{2}$ be the probability that he knows the answer, $\frac{1}{4}$ be the probability that he guesses and $\frac{1}{4}$ that he copies it. Assuming that a student, who copies the answer, will be correct with the probability $\frac{3}{4}$, what is the probability that the student knows the answer, given that he answered it correctly?

Arjun does not know the answer to one of the questions in the test. The evaluation process has negative marking. Which value would Arjun violate if he resorts to unfair means? How would an act like the above hamper his character development in the coming years?

OR

An insurance company insured 2000 cyclists, 4000 scooter drivers and 6000 motorbike drivers. The probability of an accident involving a cyclist, scooter driver and a motorbike driver are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver? Which mode of transport would you suggest to a student and why?

MARKING SCHEME

1-10	1. $\frac{7\pi}{6}$	2. $\frac{\pi}{4}$	3. Zero	4. 15	5. $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$
6. 68 units	7. 2	8. $\frac{\pi}{6}$	9. $\frac{8}{7}$	10. $\sqrt{14}$	10x1=10

SECTION - B

11. $f(x) = x^2 + x + 1$

Let $x_1, y_1 \in \mathbb{N}$ such that $f(x_1) = f(y_1)$ 1

$$\therefore x_1^2 + x_1 + 1 = y_1^2 + y_1 + 1 \Rightarrow (x_1 - y_1)(x_1 + y_1 + 1) = 0 \text{ [As } x_1 + y_1 + 1 \neq 0 \text{ for any } \mathbb{N}]$$
 1

$\Rightarrow x_1 = y_1 \Rightarrow f$ is one-one function

Clearly $f(x) = x^2 + x + 1 \geq 3$ for $x \in \mathbb{N}$ 1/2

But $f(x)$ does not assume values 1 and 2 1 1/2

$\therefore f: \mathbb{N} \rightarrow \mathbb{N}$ is not onto function

12. $\cos(\tan^{-1}x) = \cos\left(\cos^{-1}\frac{1}{\sqrt{x^2+1}}\right) = \frac{1}{\sqrt{x^2+1}}$ 1

$$\cot^{-1}\left(\frac{1}{\sqrt{x^2+1}}\right) = \sin^{-1}\left(\frac{\sqrt{x^2+1}}{\sqrt{x^2+2}}\right)$$
 1

$$\therefore \sin\left[\cot^{-1}\left(\frac{1}{\sqrt{x^2+1}}\right)\right] = \sin\left[\sin^{-1}\left(\frac{\sqrt{x^2+1}}{\sqrt{x^2+2}}\right)\right] = \frac{\sqrt{x^2+1}}{\sqrt{x^2+2}}$$
 1+1

OR

Let $x = \tan\theta$ 1/2

$$\therefore \text{LHS} = 3\sin^{-1}(\sin 2\theta) - 4\cos^{-1}(\cos 2\theta) + 2\tan^{-1}(\tan 2\theta)$$
 1 1/2

$$= 3 \times 2\theta - 8\theta + 4\theta = 2\theta = 2 \tan^{-1} x \quad 1$$

$$\Rightarrow 2 \tan^{-1} x = \frac{\pi}{3} \Rightarrow \tan^{-1} x = \frac{\pi}{6} \Rightarrow x = \frac{1}{\sqrt{3}} \quad 1$$

13.

i)

$$\begin{pmatrix} 5 & 4 & 3 \\ 4 & 3 & 5 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 11000 \\ 10700 \\ 2700 \end{pmatrix}$$

∴

$$5x + 4y + 3z = 11000$$

1 ½

$$4x + 3y + 5z = 10700$$

$$x + y + z = 2700$$

ii)

let $A = \begin{pmatrix} 5 & 4 & 3 \\ 4 & 3 & 5 \\ 1 & 1 & 1 \end{pmatrix}$

∴

$$|A| = 5(-2) - 4(-1) + 3(1)$$

1 ½

$$= -10 + 4 + 3 = -3 \neq 0$$

∴ A^{-1} exists, so equations have a unique solution.

iii) Any answer of three values which proper reasoning will be considered correct. 1

14. $x = a(\theta - \sin\theta) \Rightarrow \frac{dx}{d\theta} = a(1 - \cos\theta) = 2a \sin^2 \frac{\theta}{2}$ 1

$y = a(1 - \cos\theta) \Rightarrow \frac{dy}{d\theta} = a \sin\theta = 2a \sin \frac{\theta}{2} \cos \frac{\theta}{2}$ 1

$$\therefore \frac{dy}{dx} = \frac{2a \sin \frac{\theta}{2} \cdot \cos \frac{\theta}{2}}{2a \sin^2 \frac{\theta}{2}} = \cot \frac{\theta}{2}. \quad \frac{1}{2}$$

$$\frac{d^2y}{dx^2} = -\frac{1}{2} \operatorname{cosec}^2 \frac{\theta}{2} \cdot \frac{d\theta}{dx} = -\frac{1}{2} \frac{1}{\sin^2 \frac{\theta}{2}} \cdot \frac{1}{2a \sin^2 \frac{\theta}{2}} = \frac{-1}{4a} \operatorname{cosec}^4 \frac{\theta}{2} \quad 1\frac{1}{2}$$

$$15. y = \frac{\sin^{-1}x}{\sqrt{1-x^2}} \Rightarrow \sqrt{1-x^2} \cdot y = \sin^{-1}x \quad \frac{1}{2}$$

$$\therefore \sqrt{1-x^2} \frac{dy}{dx} + \frac{y(-2x)}{\sqrt{1-x^2}} = \frac{1}{\sqrt{1-x^2}} \quad 1$$

$$\therefore (1-x^2) \frac{dy}{dx} - xy - 1 = 0 \quad \frac{1}{2}$$

Diff. again we get

$$(1-x^2) \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} - \left[y \cdot 1 + x \frac{dy}{dx} \right] = 0 \quad 1$$

$$\Rightarrow (1-x^2) \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} - y = 0 \quad 1$$

$$16. \lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} x^2 + ax + b = 2a + b + 4$$

$$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} (3x + 2) = 8 \quad 1$$

$$\text{As } f \text{ is continuous at } x=2 \Rightarrow 2a+b+4=8 \Rightarrow 2a+b=4 \text{ --- (i)} \quad 1$$

$$\text{Similarly as } f \text{ is continuous at } x=4, 14 = 8a+5b \text{ --- (ii)} \quad 1$$

$$\text{From (i) and (ii), } a = 3, b = -2 \quad 1$$

OR

$$y = \tan^{-1} \left[\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \right] = \text{Let } x^2 = \cos 2\theta \quad 1$$

$$\therefore \tan^{-1} \left[\frac{\sqrt{1+\cos 2\theta} - \sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta} + \sqrt{1-\cos 2\theta}} \right] = \tan^{-1} \left[\frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta} \right] \quad 1$$

$$= \tan^{-1} \frac{1 - \tan \theta}{1 + \tan \theta} = \tan^{-1} \left(\tan \left(\frac{\pi}{4} - \theta \right) \right) = \frac{\pi}{4} - \theta \quad 1$$

$$\therefore y = \frac{\pi}{4} - \frac{1}{2} \cos^{-1} x^2 : \text{Let } z = \cos^{-1} x^2$$

$$\therefore y = \frac{\pi}{4} - \frac{1}{2} z \Rightarrow \frac{dy}{dz} = -\frac{1}{2} \quad \frac{1}{2} + \frac{1}{2}$$

$$17. I = \int \frac{x^3+x+1}{x^2-1} dx = \int \left[x + \frac{2x+1}{(x-1)(x+1)} \right] dx \quad 1$$

$$\text{Let } \frac{2x+1}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1} \Rightarrow A = \frac{3}{2}, B = \frac{1}{2} \quad 1$$

$$\therefore I = \int x dx + \frac{3}{2} \int \frac{dx}{x-1} + \frac{1}{2} \int \frac{dx}{x+1} = \frac{x^2}{2} + \frac{3}{2} \log(x-1) + \frac{1}{2} \log(x+1) + c \quad 1+1$$

OR

$$\int e^x \left(\frac{1-\sin x}{1-\cos x} \right) dx = \int \left[\frac{e^x [1-2\sin \frac{x}{2} \cos \frac{x}{2}]}{2\sin^2 \frac{x}{2}} \right] dx \quad 1$$

$$= \int e^x \left(\frac{1-2\sin \frac{x}{2} \cos \frac{x}{2}}{2\sin^2 \frac{x}{2}} \right) dx = \int e^x \left(\frac{1}{2} \operatorname{cosec}^2 \frac{x}{2} - \cot \frac{x}{2} \right) dx \quad \frac{1}{2}$$

$$= -\int e^x \cot \frac{x}{2} dx + \frac{1}{2} \int e^x \operatorname{cosec}^2 \frac{x}{2} dx \quad \frac{1}{2}$$

$$= -\cot \frac{x}{2} \cdot e^x - \int (e^x \operatorname{cosec}^2 \frac{x}{2} \cdot \frac{1}{2} dx) + \frac{1}{2} \int e^x \operatorname{cosec}^2 \frac{x}{2} dx \quad 1$$

$$= -e^x \cot \frac{x}{2} - \frac{1}{2} \int e^x \operatorname{cosec}^2 \frac{x}{2} dx + \frac{1}{2} \int e^x \operatorname{cosec}^2 \frac{x}{2} dx + C \quad 1$$

$$= -e^x \cot \frac{x}{2} + C$$

$$18. I = \int \frac{2x}{(x^2+1)(x^2+2)} dx$$

$$\text{Let } x^2=t, 2x dx = dt \Rightarrow I = \int \frac{dt}{(t+1)(t+2)} = \int \frac{A}{t+1} dt + \int \frac{Bdt}{t+2} \quad 1\frac{1}{2}$$

$$\text{Getting } A=1, B = -1 \quad 1$$

$$\therefore I = \int \frac{dt}{t+1} - \int \frac{dt}{t+2} = \log|t+1| - \log|t+2| + C \quad 1$$

$$= \log|x^2+1| - \log|x^2+2| + C \quad \frac{1}{2}$$

$$19. I = \int_0^{\frac{\pi}{4}} \log(1 + \tan x) dx \dots (i) \quad \int_0^{\frac{\pi}{4}} \log \left[1 + \tan \left(\frac{\pi}{4} - x \right) \right] dx, I = \int_0^{\frac{\pi}{4}} \log \left(1 + \frac{1-\tan x}{1+\tan x} \right) dx$$

1

$$\text{or } I = \int_0^{\frac{\pi}{4}} \log \left(\frac{2}{1+\tan x} \right) dx = \int_0^{\frac{\pi}{4}} [\log 2 - \log(1 + \tan x)] dx \dots (ii) \quad 1$$

Adding (i) and (ii), we get

$$\therefore 2I = \int_0^{\frac{\pi}{4}} \log 2 \, dx = \frac{\pi}{4} \log 2 \quad 1$$

$$\Rightarrow I = \frac{\pi}{8} \log 2 \quad 1$$

20. Let $\vec{d} = x\hat{i} + y\hat{j} + z\hat{k}$

As $\vec{d} \perp \vec{a}$ and $\vec{d} \perp \vec{b} \Rightarrow \vec{d} \cdot \vec{a} = 0$ and $\vec{d} \cdot \vec{b} = 0 \quad 1$

$$\vec{d} \cdot \vec{a} = 0 \Rightarrow 4x + 5y - z = 0 \text{ and } \vec{d} \cdot \vec{b} = 0 \Rightarrow x - 4y + 5z = 0 \dots (i) \quad 1$$

$$\vec{d} \cdot \vec{c} = 21 \Rightarrow 3x + y - z = 21 \dots (ii) \quad \frac{1}{2}$$

Solving (i) and (ii), we get $x=7, y=-7, z=-7 \quad 1$

$$\therefore \vec{d} = 7\hat{i} - 7\hat{j} - 7\hat{k} \quad \frac{1}{2}$$

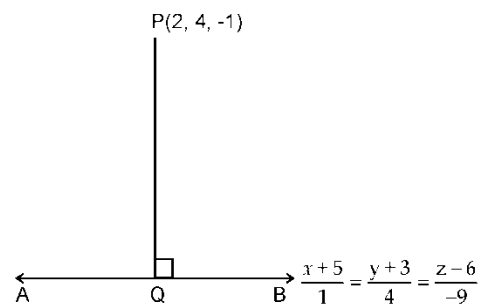
21. The equation of plane passing through $(3, -1, 2), (5, 2, 4)$ and $(-1, -1, 6)$ is

$$\begin{vmatrix} x-3 & y+1 & z-2 \\ -3+5 & 1+2 & -2+4 \\ -3-1 & -1+1 & 6-2 \end{vmatrix} = 0 \text{ or } \begin{vmatrix} x-3 & y+1 & z-2 \\ 2 & 3 & 2 \\ -4 & 0 & 4 \end{vmatrix} = 0 \quad 2$$

$$\Rightarrow 12x - 16y + 12z = 76 \text{ or } 3x - 4y + 3z = 19 \dots (i) \quad \frac{1}{2}$$

\therefore length of \perp from $(6, 5, 9)$ to (i) is

$$\frac{|18 - 20 + 27 - 19|}{\sqrt{3^2 + 4^2 + 3^2}} = \frac{6}{\sqrt{34}} \quad 1 + \frac{1}{2}$$



OR

Any point on the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9} = \lambda \quad 1$

is $(\lambda - 5, 4\lambda - 3, -9\lambda + 6)$

Let this point be Q

\therefore dr's of \overline{PQ} are $\lambda - 7, 4\lambda - 7, -9\lambda + 7 \quad \frac{1}{2}$

Since $\overline{PQ} \perp \overline{AB} \Rightarrow (\lambda - 7)1 + (4\lambda - 7)4 + (-9\lambda + 7)(-9) = 0$

$$\Rightarrow 98\lambda = 98 \Rightarrow \lambda=1 \quad 1$$

\therefore The pt. Q is $(-4, 1, -3)$ $\frac{1}{2}$

\therefore equation of PQ is $\frac{x-2}{6} = \frac{y-4}{3} = \frac{z+1}{2}$ and foot of \perp is $(-4, 1, -3)$ 1

22. Let x denote the number of non-violent persons out of selected two. X can take values

0, 1, 2 non-violent 20: Violent patriotism: 30 $\frac{1}{2}$

$$P(x = 0) = \frac{(30 \times 29)}{50 \times 49} = \frac{87}{245} \quad \frac{1}{2}$$

$$P(x = 1) = \frac{(30 \times 20 \times 2)}{50 \times 49} = \frac{120}{245} \quad \frac{1}{2}$$

$$P(x = 2) = \frac{(20 \times 19)}{50 \times 49} = \frac{38}{245} \quad \frac{1}{2}$$

$$\text{Mean} = 0 \times \frac{87}{245} + 1 \times \frac{120}{245} + 2 \times \frac{38}{245} = \frac{198}{245} \quad 1$$

Importance: In order to have a peaceful environment both the values are required patriotism and non-violence only patriotism with violence could be very dangerous 1

23. The given matrix is $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$, $|A| = -6 + 28 + 45 = 67 \neq 0$ 1

$\therefore A^{-1}$ exists

$$\text{Adj } A = \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}, \Rightarrow A^{-1} = \frac{1}{67} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix} \quad 2\frac{1}{2}$$

The given system of equations can be written as $AX=B$

$$\text{Where } A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix} \quad 1$$

$$X = A^{-1}B = \frac{1}{67} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix} \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} \quad 1$$

$$\therefore x=3, y=-2, z=1 \quad \frac{1}{2}$$

24. The given curve cuts the x-axis at $x=7$, and $y=0$ $\frac{1}{2}$

$$y = \frac{x-7}{x^2-5x+6} \Rightarrow \frac{dy}{dx} = \frac{(x^2-5x+6)-(x-7)(2x-5)}{(x^2-5x+6)^2} \quad 1\frac{1}{2}$$

$$\frac{dy}{dx} \text{ (at } x=7) = \frac{(49-35+6)-(0)}{(49-35+6)^2} = \frac{1}{20} \quad \frac{1}{2} + \frac{1}{2}$$

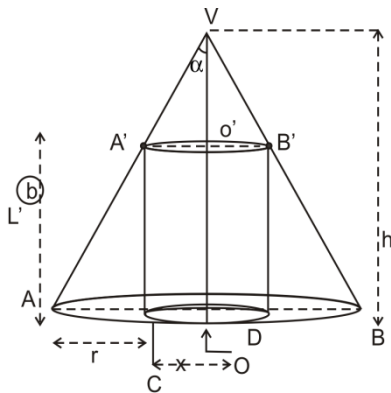
∴ Equation of tangent to the curve at (7,0) is

$$y - 0 = \frac{1}{20}(x - 7) \text{ or } x - 20y - 7 = 0 \quad 1\frac{1}{2}$$

Equation of normal to the curve at (7,0) is

$$y - 0 = -20(x - 7) = 20x + y - 140 = 0 \quad 1\frac{1}{2}$$

OR



Let $OC = x$ $\Delta VOB \sim \Delta B'DB$ 1

$$\therefore \frac{VO}{B'D} = \frac{OB}{DB} \Rightarrow B'D = \frac{h(r-x)}{r} = h'$$

Let S be the curved surface area of cylinder 1

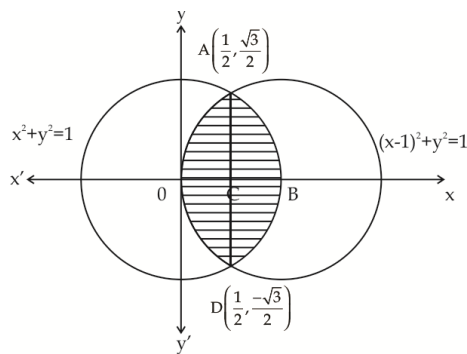
$$\therefore S = 2\pi x h' = 2\pi x \left[\frac{h(r-x)}{r} \right] = \frac{2\pi h}{r} [rx - x^2] \quad 2$$

$$\therefore \frac{ds}{dx} = \frac{2\pi h}{r} (r - 2x), \frac{d^2s}{dx^2} = -\frac{4\pi h}{r} < 0 \Rightarrow S \text{ is Maximum} \quad 1$$

$$\frac{ds}{dx} = 0 \Rightarrow r = 2x$$

∴ S is maximum when $x = \frac{r}{2}$, i.e., when radius of base of cylinder is half the radius of base of cone. 1

25. Points of inter section are $A\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ and $D\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ 1



Area of Shaded area = 2 (Area O ABCO)

$$= 2 \left[\int_0^{\frac{1}{2}} \sqrt{1 - (x-1)^2} dx + \int_{\frac{1}{2}}^1 \sqrt{1 - x^2} dx \right] \quad 1$$

$$= 2 \left[\frac{(x-1)}{2} \sqrt{1 - (x-1)^2} + \frac{1}{2} \sin^{-1} \left(\frac{x-1}{1} \right) \right]_0^{\frac{1}{2}} +$$

$$\left[\frac{x}{2} \sqrt{1-x^2} + \frac{1}{2} \sin^{-1} x \right]_{\frac{1}{2}}^1$$

2

$$= \left\{ \frac{-\sqrt{3}}{4} + \sin^{-1} \left(\frac{-1}{2} \right) - \sin^{-1}(-1) \right\} + \left\{ \sin^{-1}(1) - \frac{\sqrt{3}}{4} - \sin^{-1} \left(\frac{1}{2} \right) \right\}$$

1

$$= -\frac{\sqrt{3}}{4} - \frac{\pi}{6} + \frac{\pi}{2} + \frac{\pi}{2} - \frac{\sqrt{3}}{4} - \frac{\pi}{6}$$

$$= \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2} \right) \text{sq. units.}$$

1

26. The given diff. eqn. can be written as

$$\frac{dx}{dy} + (\cot y) x = \cos y$$

1

$$\text{I.F} = e^{\int \cot y \, dy} = e^{\log \sin y} = \sin y$$

$$\therefore \text{The solution is } x \sin y = \int \sin y \cos y \, dy + C$$

1

$$= \frac{1}{2} \int \sin 2y \, dy + c$$

$$\text{or } x \sin y = \frac{-1}{4} \cos 2y + C$$

1

It is given that $y=0$, when $x=0$

1

$$C - \frac{1}{4} = 0 \Rightarrow C = 1/4$$

$$\therefore x \sin y = \frac{1}{4} (1 - \cos 2y) = \frac{1}{2} \sin^2 y$$

1

$$\Rightarrow 2x = \sin y \text{ is the reqd. solution}$$

1

27. Here $\vec{a}_1 = 2\hat{i} + \hat{j} - 3\hat{k}$ and $\vec{a}_2 = 3\hat{i} + 3\hat{j} + 2\hat{k}$

1

$$\vec{b}_1 = \hat{i} + 2\hat{j} + 5\hat{k} \text{ and } \vec{b}_2 = 3\hat{i} - 2\hat{j} + 5\hat{k}$$

$$\vec{n} = \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 5 \\ 3 & -2 & 5 \end{vmatrix} = 20\hat{i} + 10\hat{j} - 8\hat{k}$$

1½

\therefore Vector eqn. of the reqd. plane is $(\vec{r} - \vec{a}_1) \cdot \vec{n} = 0$ or $\vec{r} \cdot \vec{n} = \vec{a}_1 \cdot \vec{n}$ or

$$\vec{r} \cdot (20\hat{i} + 10\hat{j} - 8\hat{k}) = (2\hat{i} + \hat{j} - 3\hat{k}) \cdot (20\hat{i} + 10\hat{j} - 8\hat{k}) = 40 + 10 + 24 = 74$$

2

$$\Rightarrow \vec{r} \cdot (10\hat{i} + 5\hat{j} - 4\hat{k}) = 37$$

The Cartesian equation of plane is $10x + 5y - 4z = 37$

1½

28 Suppose number of electronic operated machine = x and number of manually operated sewing machines = y

2

$\therefore x+y \leq 20$ --- (i)

and, $360x + 240y \leq 5760$ or $3x+2y \leq 48$ --- (ii)

$x \geq 0, y \geq 0$

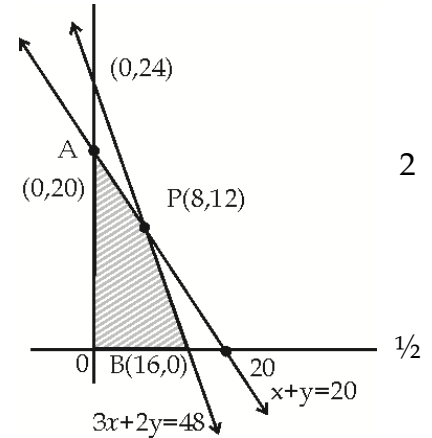
To maximise $Z=22x + 18y$

Corners of feasible region are A (0,20), P(8,12), B(16,0)

$Z_A = 18 \times 20 = 360, Z_P = 22 \times 8 + 18 \times 12 = 392, Z_B = 352$

$\therefore Z$ is maximum at $x=8$ and $y=12$

\therefore The dealer should invest in 8 electric and 12 manually operated machines 1/2



Keeping the 'save environment' factor in mind the manually operated machine should be promoted so that energy could be saved. 1

29. Let A be the event that he knows the answer, B be the event that he guesses and C be the event that he copies. 1

Then, $P(A) = 1/2, P(B) = 1/4$ and $P(C) = 1/4$

Let X be the event that he has answered correctly.

Also, $P(X/A) = 1, P(X/B) = 1/4$ and $P(X/C) = 3/4$

Thus, Required probability = $P(A/X) =$

} 1
} 1 for formula

$$\frac{P(X/A) \times P(A)}{P(X/A) \times P(A) + P(X/B) \times P(B) + P(X/C) \times P(C)}$$

$$= \frac{\frac{1}{2} \times 1}{\frac{1}{2} \times 1 + \frac{1}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{3}{4}}$$

$$= \frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{16} + \frac{3}{16}}$$

$$= \frac{2}{3}$$

1

If Arjun copies the answer, he will be violating the value of honesty in his character. He should not guess the answer as well as that may fetch him negative marking for a wrong guess. He should accept the question the way it is and leave it

unanswered as cheating may get him marks in this exam but this habit may not let him develop an integrity of character in the long run. 2

OR

Let the events defined are E1: Person chosen is a cyclist

E2: Person chosen is a scooter-driver

E3: Person chosen is a motorbike driver

A: Person meets with an Accident 1/2

$P(E1) = 1/6, P(E2) = 1/3, P(E3) = 1/2$ 1

$P(A/E1) = 0.01, P(A/E2) = 0.03, P(A/E3) = 0.15$ 2 1

$P(E1/A) = P(A/E1) \cdot P(E1) / P(A)$ 1

$= 1/2$ 1/2

Suggestion: Cycle should be promoted as it is good for 1/2

- i. **Health** 1/2
- ii. **No pollution** 1/2
- iii. **Saves energy(no petrol)** 1/2